

SETHU BHASKARA MATRIC HIGHER SECONDARY
SCHOOL.
QUARTERLY EXAMINATION - MATHEMATICS.
CLASS: XII.

PART. I.

1 d) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

11 d) $\pi/2$

2 a) 11

12 c) Unique solution.

3 b) 1

13 d) $2/\sqrt{3}$

4 b) $\frac{1}{2} |z|^2$

14 a) $x+2y+3=0$

5 c) 1

15 d) $(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}})$

6 b) 1

16 c) 0°

7 b) $-9/r$

17 d) 0

8 c) 2

18 c) $\frac{\sqrt{7}}{2\sqrt{2}}$

9 d) n

19 a) $\frac{18}{5}$

10 d) $\pi/2 - x$.

20 b) -1.

PART II.

21
$$\begin{aligned} 2x - y &= 8 \\ 3x + 2y &= -2 \end{aligned}$$

$$\begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ -2 \end{pmatrix}$$

$$X = A^{-1}B$$

$$|A| = 7 \neq 0 \therefore A^{-1} \text{ exists.}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj} A = \frac{1}{7} \begin{pmatrix} 2 & 1 \\ -3 & 2 \end{pmatrix}$$

$$X = \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 14 \\ -28 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -4 \end{pmatrix} \quad \begin{aligned} x &= 2, \\ y &= -4 \end{aligned}$$

(22) $i^{1729} = i^{1728} \cdot i^1$
 $= i$

(23) $z = x + iy$
 $i\bar{z} = i(x - iy)$
 $= xi - i^2 y$
 $= y + ix$

$$\text{Re}(i\bar{z}) = y.$$

$$23 \quad S_1 = 2 + \frac{1}{2} + 1 = \frac{7}{2}$$

$$S_2 = 2(\frac{1}{2}) + \frac{1}{2}(1) + 1 \times 2$$

$$= \frac{7}{2}.$$

$$S_3 = 2(\frac{1}{2})(1) = 1.$$

$$x^3 - \frac{7}{2}x^2 + \frac{7}{2}x - 1 = 0$$

$$2x^3 - 7x^2 + 7x - 2 = 0$$

$$25 \quad \sin^{-1}(\sin(5\pi/6))$$

$$= \sin^{-1}(\sin(\pi - \pi/6))$$

$$= \sin^{-1}(\sin \pi/6)$$

$$= \pi/6 \in [-\pi/2, \pi/2]$$

$$26 \quad -1 \leq \frac{2 + \sin x}{3} \leq 1$$

$$-5 \leq \sin x \leq 1$$

$$-1 \leq \sin x \leq 1$$

$$-\sin^{-1}(1) \leq x \leq \sin^{-1}(1)$$

$$-\pi/2 \leq x \leq \pi/2$$

The domain of

$$\cos^{-1}\left(\frac{2 + \sin x}{3}\right) = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$27 \quad x^2 + y^2 = 16, \quad a^2 = 16.$$

$$y = 2\sqrt{2}x + c.$$

$$m = 2\sqrt{2}, \quad c = c.$$

$$c^2 = a^2(1 + m^2)$$

$$c^2 = 16(1 + 8)$$

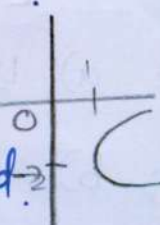
$$c^2 = 16 \times 9,$$

$$c = \pm 4 \times 3 = \pm 12.$$

$$(28) \quad V(1, -2), F(4, -2)$$

$$a = \sqrt{(4-1)^2} = 3$$

It is open rightward.



$$(y - k)^2 = 4a(x - h)$$

$$(y + 2)^2 = 12(x - 1)$$

$$(29) \quad \vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$$

$$\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}.$$

Volume of parallelepiped

$$= |[\vec{a}, \vec{b}, \vec{c}]|$$

$$= \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = |-7|$$

$$= 7.$$

$$(30) \quad \vec{r} \cdot (6\hat{i} + 4\hat{j} - 3\hat{k}) = 12$$

$$6x + 4y - 3z = 12$$

$$\frac{6x}{12} + \frac{4y}{12} + \frac{(-3z)}{12} = 1$$

$$\frac{x}{2} + \frac{y}{3} + \frac{z}{-4} = 1$$

Part III.

31

$$A = \begin{bmatrix} 0 & 5 \\ -1 & 6 \end{bmatrix}$$

$$[A/I_2] = \left[\begin{array}{cc|cc} 0 & 5 & 1 & 0 \\ -1 & 6 & 0 & 1 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{cc|cc} -1 & 6 & 0 & 1 \\ 0 & 5 & 1 & 0 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 6/5 & -1 \\ 0 & 1 & 1/5 & 0 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} 6/5 & -1 \\ 1/5 & 0 \end{bmatrix}$$

32. $\text{adj } A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

$$\text{adj}(\text{adj } A) = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

33. $|z+i| = |z-1|$

$$|x+iy+i| = |x+iy-1|$$

$$\Rightarrow |x+i(y+1)| = |x-1+iy|$$

$$\sqrt{x^2+(y+1)^2} = \sqrt{(x-1)^2+y^2}$$

Squaring

$$x^2+y^2+2y+1 = x^2-2x+1+y^2$$

$$2x+2y=0$$

$$x+y=0.$$

34

$$x^4 - 14x^2 + 45 = 0$$

$$\text{Let } y = x^2$$

$$y^2 - 14y + 45 = 0$$

$$(y-9)(y-5) = 0$$

$$y=9, y=5$$

$$x = \pm 3, x = \pm \sqrt{5}.$$

35

$$\tan^{-1}(-1) + \cos^{-1}(1/2) + \sin^{-1}(-1/2)$$

$$= -\pi/4 + \pi/3 - \pi/6$$

$$= \frac{-3\pi + 4\pi - 2\pi}{12}$$

$$= -\pi/12.$$

36

$$\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$$

$$\vec{n} = 6\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\sin \theta = \frac{\vec{b} \cdot \vec{n}}{|\vec{b}| |\vec{n}|}$$

$$\vec{b} \cdot \vec{n} = (\hat{i} + 2\hat{j} - 2\hat{k}) \cdot$$

$$(6\hat{i} + 3\hat{j} + 2\hat{k})$$

$$= 1(6) + 2(3) + (-2)(2)$$

$$= 6 + 6 - 4 = 8$$

$$|\vec{b}| = 3, |\vec{n}| = 7$$

$$\sin \theta = 8/21$$

$$\theta = \sin^{-1}(8/21)$$

37 Equation of the ellipse is $x^2 + 4y^2 = 32$, $\theta = \pi/4$

$$\frac{x^2}{32} + \frac{4y^2}{32} = 1$$

$$a = 8\sqrt{2} \quad b = 2\sqrt{2}$$

$$(x_1, y_1) = (a \cos \theta, b \sin \theta) \\ = (8, 2)$$

Equation of tangent

$$xx_1 + 4yy_1 = 32$$

$$x + y = 4$$

Equation of normal

$$x - y = 6$$

38 $y^2 = 4(1.2)x$

$$y^2 = 4.8x$$

It passes through $(x_1, 2.5)$

$$(2.5)^2 = 4.8x_1$$

$$x_1 = 1.3m$$

39 $A(2, 3, 4), B(-1, 4, 5)$

$$C(8, 1, 2)$$

Direction ratios of line

$$\text{joining } A \text{ \& } B = (-3, 1, 1)$$

Direction ratios of line

$$\text{joining } B \text{ \& } C = (9, -3, 3)$$

$$\frac{b_1}{d_1} = -3/9 = -1/3$$

$$b_2/d_2 = -1/3, b_3/d_3 = -1/3$$

$$b_1/d_1 = b_2/d_2 = b_3/d_3$$

$AB \parallel BC$, B is common point $\therefore A, B, C$ are collinear

40. $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \alpha$

$$\cot \alpha = \frac{1}{\sqrt{1-x^2}} \text{ and } \alpha \text{ is}$$

acute

$$\sec \alpha = \frac{x}{1}, \text{ thus } \alpha = \sec^{-1} x$$

$$\text{Hence } \cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \sec^{-1} x$$

$$|x| > 1$$

Part IV

$$AX = B$$

41 a)

$$\begin{bmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 3 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 9 \\ 1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$|A| = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 3 & -1 & -1 \end{vmatrix} = 16$$

Cofactor of A

$$= \begin{bmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{1}{16} \begin{bmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{bmatrix} \begin{bmatrix} 9 \\ 9 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{16} \begin{bmatrix} 40 \\ 42 \\ 62 \end{bmatrix}$$

$$41) \quad b) \quad [A/B] = \begin{bmatrix} 1 & -1 & 2 & 2 \\ 2 & 1 & 4 & 7 \\ 4 & -1 & 1 & 4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 2 & 2 \\ 0 & 3 & 0 & 3 \\ 0 & 3 & -7 & -4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 2 & 2 \\ 0 & 3 & 0 & 3 \\ 0 & 0 & -7 & -7 \end{bmatrix}$$

$$\rho(A) = \rho(A/B) = 3 = \text{No. of unknowns}$$

$\therefore$ System has a unique solution.

$$-7z = -7 \Rightarrow z = 1$$

$$y = 1, x = 1$$

$$42) \quad a) \quad \arg\left(\frac{z-i}{z+i}\right) = \arg(z-i) - \arg(z+i)$$

$$= \arg(x+i(y-1)) - \arg(x+2+iy)$$

$$= \tan^{-1}\left(\frac{y-1}{x}\right) - \tan^{-1}\left(\frac{y}{x+2}\right)$$

$$= \tan^{-1}\left(\frac{(x+2)(y-1)-yx}{x(x+2)+y(y-1)}\right)$$

$$\arg\left(\frac{z-i}{z+i}\right) = \pi/4$$

$$\Rightarrow (x+2)(y-1) - yx$$

$$= x(x+2) + y(y-1)$$

$$\Rightarrow x^2 + y^2 + 3x - 3y + 2 = 0$$

$$42) \quad b) \quad z = \left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$$

$$\frac{19+9i}{5-3i} = \frac{(19+9i)(5+3i)}{(5-3i)(5+3i)}$$

$$= (2+3i)$$

$$\frac{8+i}{1+2i} = \frac{(8+i)(1-2i)}{(1+2i)(1-2i)}$$

$$= 2-3i$$

$$\therefore z = (2+3i)^{15} - (2-3i)^{15}$$

$$\bar{z} = (2+3i)^{15} - (2-3i)^{15}$$

$$= -z$$

$\therefore z$ is purely imaginary

$$43) \quad 6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$$

$$a) \quad 6(x^4+1) - 35(x^3+x) + 62x^2 = 0$$

$$x^2 \left[6\left(x^2 + \frac{1}{x^2}\right) - 35\left(x + \frac{1}{x}\right) + 62 \right] = 0$$

$$6\left(x + \frac{1}{x}\right)^2 - 35\left(x + \frac{1}{x}\right) + 50 = 0$$

$$6y^2 - 35y + 50 = 0.$$

$$3y[2y-5] - 10[2y-5] = 0$$

$$(3y-10)(2y-5) = 0$$

$$y = 10/3, \text{ or } 5/2.$$

$$\text{When } y = 5/2$$

$$x + \frac{1}{x} = 5/2,$$

$$\therefore x = 1/2, 2.$$

$$\text{When } y = 10/3.$$

$$x + \frac{1}{x} = 10/3$$

$$x = 1/3, 3$$

$$\therefore \text{roots are } 1/2, 1/3,$$

$$2, 3$$

43
b)

$$\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) =$$

$$\tan^{-1}\left(\frac{a_2-a_1}{1+a_1a_2}\right)$$

$$= \tan^{-1}a_2 - \tan^{-1}a_1$$

$$\tan^{-1}\left(\frac{d}{1+a_2a_3}\right) =$$

$$\tan^{-1}a_3 - \tan^{-1}a_2$$

$$\tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right) =$$

$$\tan^{-1}a_n - \tan^{-1}a_{n-1}.$$

$$\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right)$$

$$+ \dots + \tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)$$

$$= \tan^{-1}a_n - \tan^{-1}a_1$$

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \dots + \tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right]$$

$$= \tan\left[\tan^{-1}a_n - \tan^{-1}a_1\right]$$

$$= \tan\left[\tan^{-1}\left(\frac{a_n-a_1}{1+a_1a_n}\right)\right]$$

$$= \frac{a_n-a_1}{1+a_1a_n}$$

$$= \frac{a_n-a_1}{1+a_1a_n}$$

44
a)

Condition for line

$y = mx + c$ to be a tangent to the ellipse

$$\text{is } c^2 = a^2m^2 + b^2.$$

Equation of ellipse is

$$x^2 + 3y^2 = 12.$$

$$\frac{x^2}{12} + \frac{y^2}{4} = 1. \text{ and}$$

line is $y = x + 4$

$$a^2 = 12, b^2 = 4, m = 1, c = 4$$

$$c^2 = a^2 m^2 + b^2$$

$$a^2 m^2 + 16 = 12(1) + 4$$

a tangent to the ellipse
Point of Contact is

$$\left(-\frac{a^2 m}{c}, \frac{b^2}{c}\right) = (-3, 1)$$

44

b) Equation of ellipse

$$\text{is } 4x^2 + 24x + y^2 - 2y + 21 = 0$$

$$4(x^2 + 6x + 9 - 9) +$$

$$(y^2 - 2y + 1 - 1) + 21 = 0$$

$$4(x+3)^2 - 36 + (y-1)^2 - 1 + 21 = 0$$

$$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{16} = 1$$

Centre is $(-3, 1)$,

$a = 4, b = 2$, and

major is parallel to y axis

$$c^2 = 16 - 4 = 12$$

$$c = \pm 2\sqrt{3}$$

$$\text{foci} = (-3, 2\sqrt{3} + 1) \neq$$

$$(-3, -2\sqrt{3} + 1)$$

$$\text{vertices } (3, \pm 4 + 1)$$

$$= (3, 5) \neq (3, -3)$$

length of latus

$$\text{rectum} = \frac{2b^2}{a} = 2 \text{ units}$$

45

a)

$$\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$$

$$CQ = y, CP = \frac{1}{2}y$$

$$y + \frac{1}{2}y = 150$$

$$y = 100$$

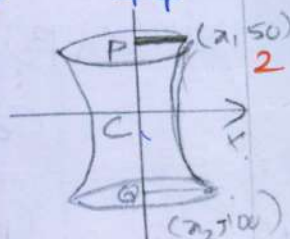
$$(x_1, 50) \Rightarrow \frac{x_1^2}{30^2} - \frac{50^2}{44^2} = 1$$

$$x_1 = 45.41$$

$$\text{At } (x_2, 100),$$

$$\frac{x_2^2}{30^2} - \frac{100^2}{44^2} = 1$$

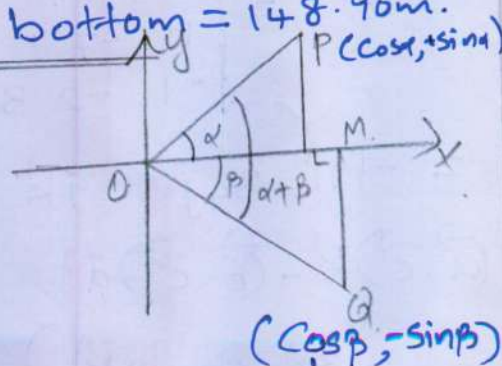
$$x_2 = 74.45$$



Diameter of top = 90.92m

bottom = 148.90m

45
b)



$$\vec{OP} = \cos \alpha \hat{i} + \sin \alpha \hat{j}$$

$$\vec{OQ} = \cos \beta \hat{i} - \sin \beta \hat{j}$$

$$\vec{OQ} \times \vec{OP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos \beta \sin \alpha & 0 & 0 \\ \cos \alpha \sin \beta & 0 & 0 \end{vmatrix}$$

$$= \hat{k} (\sin \alpha \cos \beta + \cos \alpha \sin \beta) \rightarrow (1)$$

By definition

$$\begin{aligned} \vec{OQ} \times \vec{OP} &= |\vec{OQ}| |\vec{OP}| \sin(\alpha + \beta) \hat{k} \\ &= \sin(\alpha + \beta) \hat{k} \rightarrow (2) \end{aligned}$$

From (1) + (2)

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta.$$

$$46 \quad a) \quad \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 3 & 5 & 2 \end{vmatrix}$$

$$= 11\hat{i} - 7\hat{j} + \hat{k}$$

$$(\vec{a} \times \vec{b}) \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 11 & -7 & 1 \\ -1 & -2 & 3 \end{vmatrix}$$

$$= -19\hat{i} - 34\hat{j} - 29\hat{k}$$

$$(\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}$$

$$= -11(3\hat{i} + 5\hat{j} + 2\hat{k})$$

$$- (-7)(2\hat{i} + 3\hat{j} - \hat{k})$$

$$= -19\hat{i} - 34\hat{j} - 29\hat{k}$$

$$(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}$$

46
b) Vector form

$$\vec{r} = \vec{a} + t\vec{b}$$

$$\vec{a} = 4\hat{i} + 2\hat{j} - 3\hat{k}$$

$$\vec{b} = -4\hat{i} - 2\hat{j} + \frac{3}{2}\hat{k}$$

$$= -\frac{1}{2}(8\hat{i} + 4\hat{j} - 3\hat{k})$$

$$\vec{r} = (4\hat{i} + 2\hat{j} - 3\hat{k}) + t(8\hat{i} + 4\hat{j} - 3\hat{k})$$

Cartesian form

$$\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3}$$

$$(x_1, y_1, z_1) = (-4, 2, -3)$$

$$(b_1, b_2, b_3) = (8, 4, -3)$$

$$\frac{x+4}{8} = \frac{y-2}{4} = \frac{z+3}{-3} \text{ is}$$

the required equation

$$47 \quad a) \quad \vec{u} = 5\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\vec{v} = \hat{i} + \hat{j} - \hat{k}, p = 9.$$

$$\vec{v} = \vec{u} + 2 \frac{[p - \vec{u} \cdot \vec{v}]}{|\vec{v}|^2} \vec{v}$$

$$= 5\hat{i} + 2\hat{j} + 6\hat{k} + 2 \frac{(9 - (5+2-6))}{3} (\hat{i} + \hat{j} - \hat{k})$$

$$= (5\hat{i} + 2\hat{j} + 6\hat{k}) + \frac{2}{3} \times 8 (\hat{i} + \hat{j} - \hat{k})$$

$$\begin{aligned}
 & 5\hat{i} + 2\hat{j} + 6\hat{k} + \frac{16\hat{i} + 16\hat{j} - 16\hat{k}}{3} \\
 &= \frac{15\hat{i} + 6\hat{j} + 18\hat{k} + 16\hat{i} + 16\hat{j} - 16\hat{k}}{3} \\
 &= \frac{31\hat{i} + 22\hat{j} + 2\hat{k}}{3}
 \end{aligned}$$

47

b) $(x-4)(x+1)(x-2)(x-1) + 8 = 0$

$$\begin{aligned}
 & (x^2 - 4x + x - 4) \\
 & (x^2 - 2x - x + 2) + 8 = 0
 \end{aligned}$$

$$\begin{aligned}
 & (x^2 - 3x - 4)(x^2 - 3x + 2) \\
 & + 8 = 0
 \end{aligned}$$

Let $y = x^2 - 3x$

$$(y-4)(y+2) + 8 = 0$$

$$y^2 + 4y + 2y - 8 + 8 = 0$$

$$y^2 - 2y = 0$$

$$y = 0, y - 2 = 0.$$

$$x^2 - 3x = 0, x^2 - 3x - 2 = 0$$

$$x = 0, 3.$$

$$x^2 - 3x - 2 = 0$$

$$x = \frac{3 \pm \sqrt{9+8}}{2}$$

$$x = \frac{3 \pm \sqrt{17}}{2}$$

$\therefore$ roots are 0, 3,

$$\frac{3 \pm \sqrt{17}}{2}$$

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